General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2010 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1(a)	$y_1 = 2 + 0.1\times\left[3\ln(2\times3+2)\right] = 2 + 0.3\ln8$ $= 2.6238(3...)$	M1A1		
	$y(3.1) = 2.6238$ (to 4dp)	A1	3	Condone greater accuracy
(b)	$k_1 = 0.1\times3\ln8 = 0.6238(32\ldots)$	B1F		PI ft from (a), 4dp or better
	$k_2 = 0.1\times\mathrm{f}(3.1,\ 2.6238(32...))$	M1		
	$\ldots = 0.1\times3.1\times\ln8.8238(32..)$ $[= 0.6750(1\ldots)$	A1F		PI; ft on $0.1\times3.1\times\ln[6.2 + \text{answer(a)}]$
	$y(3.1) = 2 + \frac{1}{2}\left[0.6238(3..) + 0.6750(1..)\right]$	m1		
	$= 2.6494(2\ldots) = 2.6494$ to 4dp	A1	5	CAO Must be 2.6494
	Total		**8**	
2(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4+3x}\times3$	M1		Chain rule
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -3(4+3x)^{-2}\times3 = -9(4+3x)^{-2}$	M1A1	3	M1 for quotient (PI) or chain rule used
(b)	$\ln(4+3x) = \ln4 + y'(0)\,x + y''(0)\frac{1}{2}x^2 + ..$	M1		Clear attempt to use Maclaurin's theorem with numerical values for $y'(0)$ and $y''(0)$
	First three terms: $\ln4 + \frac{3}{4}x - \frac{9}{32}x^2$	A1F	2	ft on c's answers to (a) provided $y'(0)$ and $y''(0)$ are $\neq 0$. Accept 1.38(6..) for ln4
(c)	$\ln(4-3x) = \ln4 - \frac{3}{4}x - \frac{9}{32}x^2$	B1F	1	ft $x \to -x$ in c's answer to (b)
(d)	$\ln\left(\frac{4+3x}{4-3x}\right) = \ln(4+3x) - \ln(4-3x)$	M1		
	$\approx \ln4 + \frac{3}{4}x - \frac{9}{32}x^2 - \ln4 + \frac{3}{4}x + \frac{9}{32}x^2$			
	$\approx \frac{3}{2}x$	A1	2	CSO AG
	Total		**8**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$u = \frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	M1		
	$x\frac{\mathrm{d}u}{\mathrm{d}x} + 2u = 3x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} + \frac{2}{x}u = 3$	A1	2	CSO AG Substitution into LHS of DE and completion
3(b)	IF is exp $(\int \frac{2}{x}\mathrm{d}x)$	M1		exp $(\int \frac{k}{x}\mathrm{d}x)$, for $k = \pm 2, \pm 1$ and integration attempted
	$= \mathrm{e}^{2\ln x}$; $= x^2$	A1;A1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(ux^2\right) = 3x^2$	M1		LHS as differential of $u \times$ IF
	$ux^2 = x^3 + A \Rightarrow u = x + Ax^{-2}$	A1	5	Must have an arbitrary constant
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = x + Ax^{-2}$	M1		and with integration attempted
	$\frac{\mathrm{d}y}{\mathrm{d}x} = x + Ax^{-2} \Rightarrow y = \frac{1}{2}x^2 - \frac{A}{x} + B$	A1F	2	ft only if IF is M1A0A0
	Total		**9**	
4(a)	$\sin 3x = 3x - \frac{1}{3!}(3x)^3 + = 3x - 4.5x^3 + \ldots$	B1	1	
(b)	$\cos 2x = 1 - \frac{1}{2!}(2x)^2 + \ldots.$	B1		
	$\lim_{x \to 0}\left[\frac{3x\cos 2x - \sin 3x}{5x^3}\right] = \lim_{x \to 0}\frac{3x - 6x^3 - 3x + 4.5x^3 + \ldots}{5x^3}$	M1		Using expansions
	$= \lim_{x \to 0}\frac{-1.5 + (o(x^2))\ldots}{5}$	m1		Division by x^3 stage to reach relevant form of quotient before taking limit.
	$= -\frac{3}{10}$	A1	4	CSO OE
	Total		**5**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$y_{PI} = pxe^{-2x} \Rightarrow \dfrac{dy}{dx} = pe^{-2x} - 2pxe^{-2x}$	M1		Product Rule used
	$\Rightarrow \dfrac{d^2y}{dx^2} = -2pe^{-2x} - 2pe^{-2x} + 4pxe^{-2x}$	A1		
	$-4pe^{-2x} + 4pxe^{-2x} + 3pe^{-2x} - 6pxe^{-2x} + 2pxe^{-2x} = 2e^{-2x}$.	M1		Sub. into DE
	$-pe^{-2x} = 2e^{-2x} \Rightarrow p = -2$	A1F	4	ft one slip in differentiation
5(b)	Aux. eqn. $m^2 + 3m + 2 = 0$ $\Rightarrow m = -1, -2$	B1		
	CF is $Ae^{-x} + Be^{-2x}$	M1		ft on real values of m only
	GS $y = Ae^{-x} + Be^{-2x} - 2xe^{-2x}$.	B1F		Their CF + their PI must have 2 arb consts
	When $x = 0$, $y = 2 \Rightarrow A + B = 2$	B1F		Must be using GS; ft on wrong non-zero values for p and m
	$\dfrac{dy}{dx} = -Ae^{-x} - 2Be^{-2x} - 2e^{-2x} + 4xe^{-2x}$	B1F		Must be using GS; ft on wrong non-zero values for p and m
	When $x = 0$, $\dfrac{dy}{dx} = 0 \Rightarrow -A - 2B - 2 = 0$	B1F		Must be using GS; ft on wrong non-zero values for p and m and slips in finding $y'(x)$
	Solving simultaneously, 2 eqns each in two arbitrary constants	m1		
	$A = 6$, $B = -4$; $y = 6e^{-x} - 4e^{-2x} - 2xe^{-2x}$.	A1	8	CSO
	Total		**12**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	The interval of integration is infinite	E1	1	OE
(b)(i)	$x=\frac{1}{y} \Rightarrow$ '$\mathrm{d}x=-y^{-2}\,\mathrm{d}y$'			
	$\int \frac{\ln x^2}{x^3}\,\mathrm{d}x \Rightarrow \int (y^3 \ln y^{-2})(-y^{-2})\mathrm{d}y$	M1		
	$=\int -y\ln y^{-2}\,\mathrm{d}y = \int 2y\ln y\,\mathrm{d}y$	A1	2	CSO AG
(ii)	$\int 2y\ln y\,\mathrm{d}y = y^2\ln y - \int y^2\left(\frac{1}{y}\right)\mathrm{d}y$	M1		$...= ky^2 \ln y \pm \int \mathrm{f}(y)\,\mathrm{d}y$ with f(y) not involving the 'original' ln y
		A1		
	$\ldots\ldots = y^2\ln y - \frac{1}{2}y^2 + c$	A1		Condone absence of '+ c'
	$\int_0^1 2y\ln y\,\mathrm{d}y = \lim_{a\to 0}\int_a^1 2y\ln y\,\mathrm{d}y$			
	$=\left(0-\frac{1}{2}\right)-\lim_{a\to 0}\left[a^2\ln a - \frac{a^2}{2}\right]$	M1		
	$=-\frac{1}{2}$ since $\lim_{a\to 0} a^2\ln a = 0$	A1	5	CSO Must see clear indication that cand has correctly considered $\lim_{a\to 0} a^k \ln a = 0$
(iii)	So $\int_1^\infty \frac{\ln x^2}{x^3}\,\mathrm{d}x = \frac{1}{2}$	B1F	1	ft on minus c's value as answer to (b)(ii)
	Total		**9**	
7	Aux. eqn. $m^2+4=0 \Rightarrow m=\pm 2\mathrm{i}$	B1		
	CF is $A\cos 2x + B\sin 2x$	M1		OE. If m is real give M0
		A1F		ft on incorrect complex value for m
	PI: Try ax^2+b	M1		Award even if extra terms, provided the relevant coefficients are shown to be zero.
	$+ c\sin x$	M1		
	$2a - c\sin x + 4ax^2 + 4b + 4c\sin x = 8x^2 + 9\sin x$			
	$a=2,\ b=-1,$	A1		Dep on relevant M mark
	$c=3$	A1		Dep on relevant M mark
	$(y =)\ A\cos 2x + B\sin 2x + 2x^2 - 1 + 3\sin x$	B1F	8	Their CF + their PI. Must be exactly two arbitrary constants
	Total		**8**	

Q	Solution	Marks	Total	Comments
8(a)	$4\sin\theta(1-\sin\theta)=1$	M1		Elimination of r or θ { $r = 4[1-(1/r)]$}
	$4\sin^2\theta - 4\sin\theta + 1 = 0$	A1		{ $r^2 - 4r + 4 = 0$ }
	$(2\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 0.5$	m1		Valid method to solve quadratic eqn. PI { $(r-2)^2 = 0 \Rightarrow r = 2$ }
	$\theta = \frac{\pi}{6}$, $\theta = \frac{5\pi}{6}$, $r = 2$	A2,1		A1 for any two of the three.
	[$P\left(2, \frac{\pi}{6}\right)$ $Q\left(2, \frac{5\pi}{6}\right)$]		5	SC: Verification of $P\left(2, \frac{\pi}{6}\right)$ scores max of B1 & a further B1 if $Q\left(2, \frac{5\pi}{6}\right)$ stated
8(b)	Area triangle OPQ = $\frac{1}{2}\times 2\times r_Q \times \sin POQ$	M1		Any valid method to correct (ft eg on r_Q) expression with just one remaining unknown
	Angle $POQ = \frac{5\pi}{6} - \frac{\pi}{6}$ $\left(=\frac{2\pi}{3}\right)$	m1		Valid method to find remaining unknown either relevant angle or relevant side
	Area triangle $OPQ = 2\sin\frac{2\pi}{3} = \sqrt{3}$	A1		
	Unshaded area bounded by line OP and arc OP = $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [4(1-\sin\theta)]^2\, d\theta$	M1		Use of $\frac{1}{2}\int r^2\, d\theta$ for relevant area(s) (condone missing/wrong limits)
	$= 8\int (1 - 2\sin\theta + \sin^2\theta)\, d\theta$	B1		Correct expn of $(1-\sin\theta)^2$
	$= 8\int \left(1 - 2\sin\theta + \frac{1-\cos 2\theta}{2}\right) d\theta$	M1		Attempt to write $\sin^2\theta$ in terms of $\cos 2\theta$
	$= 8\left[\theta + 2\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]$ (+ c)	A1F		Correct integration ft wrong coeffs
	$8\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1-\sin\theta)^2\, d\theta =$ $8\times\left[\frac{3\theta}{2} + 2\cos\theta - \frac{\sin 2\theta}{4}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$			
	$= 8\times\{\frac{3\pi}{4} - \left(\frac{3\pi}{12} + 2\cos\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{6}\right)\}$	m1		$F\left(\frac{\pi}{2}\right) - F\left(\frac{\pi}{6}\right)$ OE for relevant area(s)
	$= 8\times\left(\frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8}\right)$ $\{= 4\pi - 7\sqrt{3}\}$	A1F		ft one slip; accept terms in π and $\sqrt{3}$ left unsimplified
	Shaded area = Area of triangle OPQ − $2\times\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} [4(1-\sin\theta)]^2\, d\theta$	M1		OE
	Shaded area = $\sqrt{3} - 16\left(\frac{\pi}{2} - \sqrt{3} + \frac{\sqrt{3}}{8}\right) = 15\sqrt{3} - 8\pi$	A1	11	CSO Accept $m = 15$, $n = -8$
	Total		**16**	
	TOTAL		**75**	